Gacha Game Analysis and Design

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Motivation

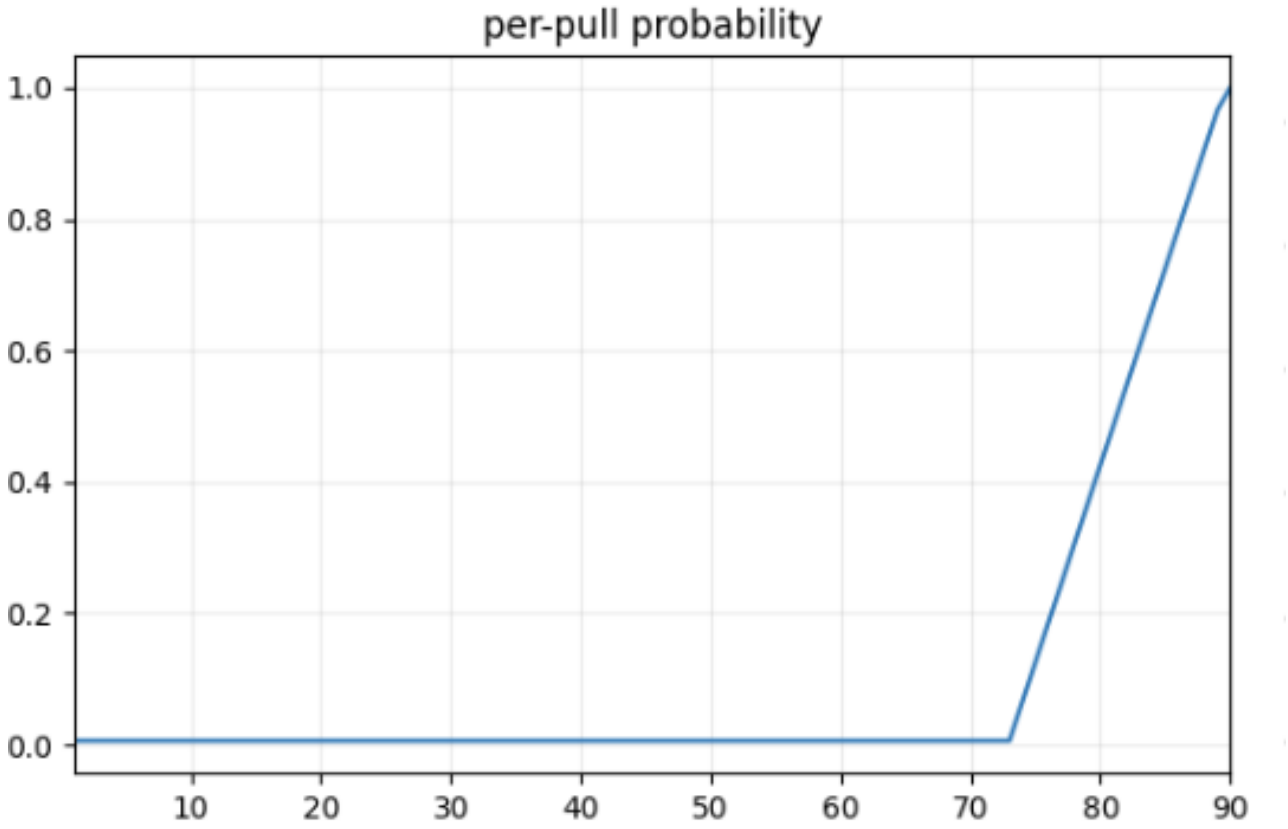
What is Gacha?

A game model consisting of small probabilities to give players rare items, similar to capsule machines

Games with Gacha Mechanisms

Genshin Impact, Arknights, OverWatch, Clash Royale, and many others

Motivation


per-pull probability
1.0
0.8
0.6
0.4
0.2
0.0
10
20
30
40
50
60
70
80
90


What is Gacha?
A game model consisting of small probabilities to give players rare items, similar to capsule machines
Games with Gacha Mechanisms
Genshin Impact, Arknights, OverWatch, Clash Royale, and many others

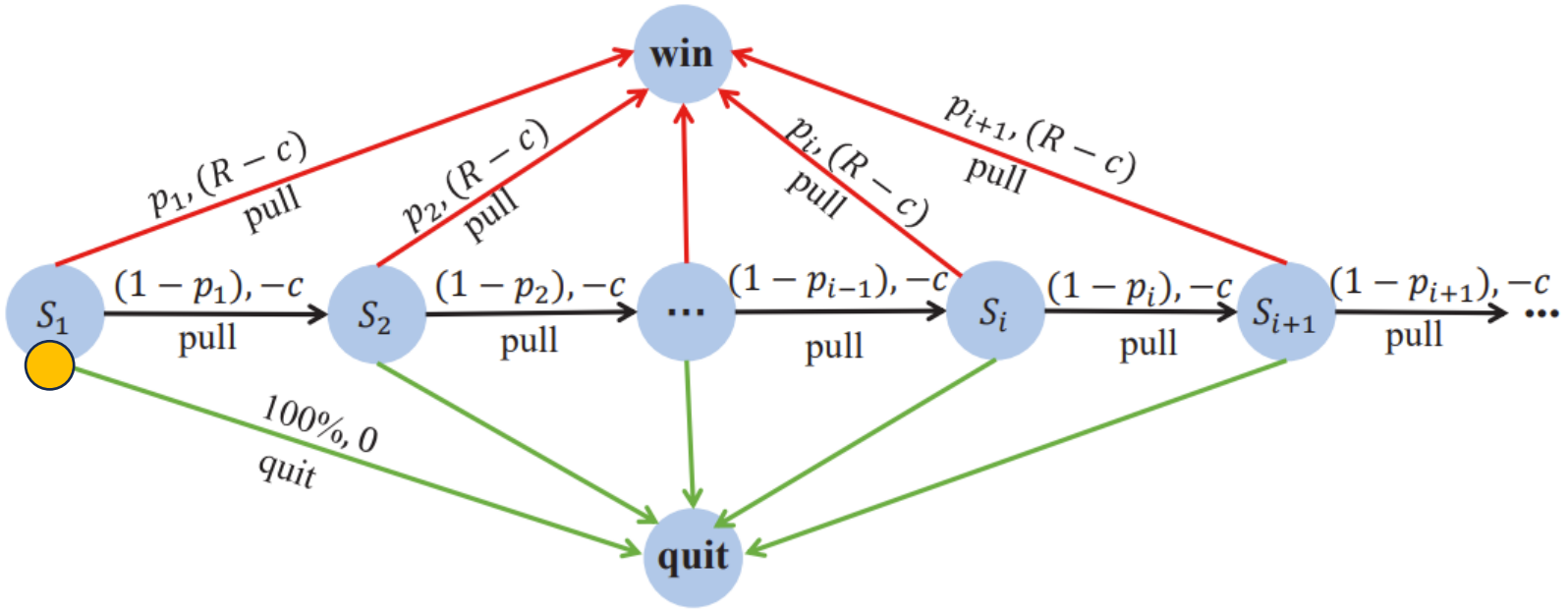
What is Gacha Game

- Gacha game is a special probabilistic selling strategy.
- The seller is selling gacha pulls to the buyer.
- Each gacha pull provides a certain probability to win.
- Once the buyer wins, the buyer will receive the game reward.
- The probability can be varied.

Gacha Game Model

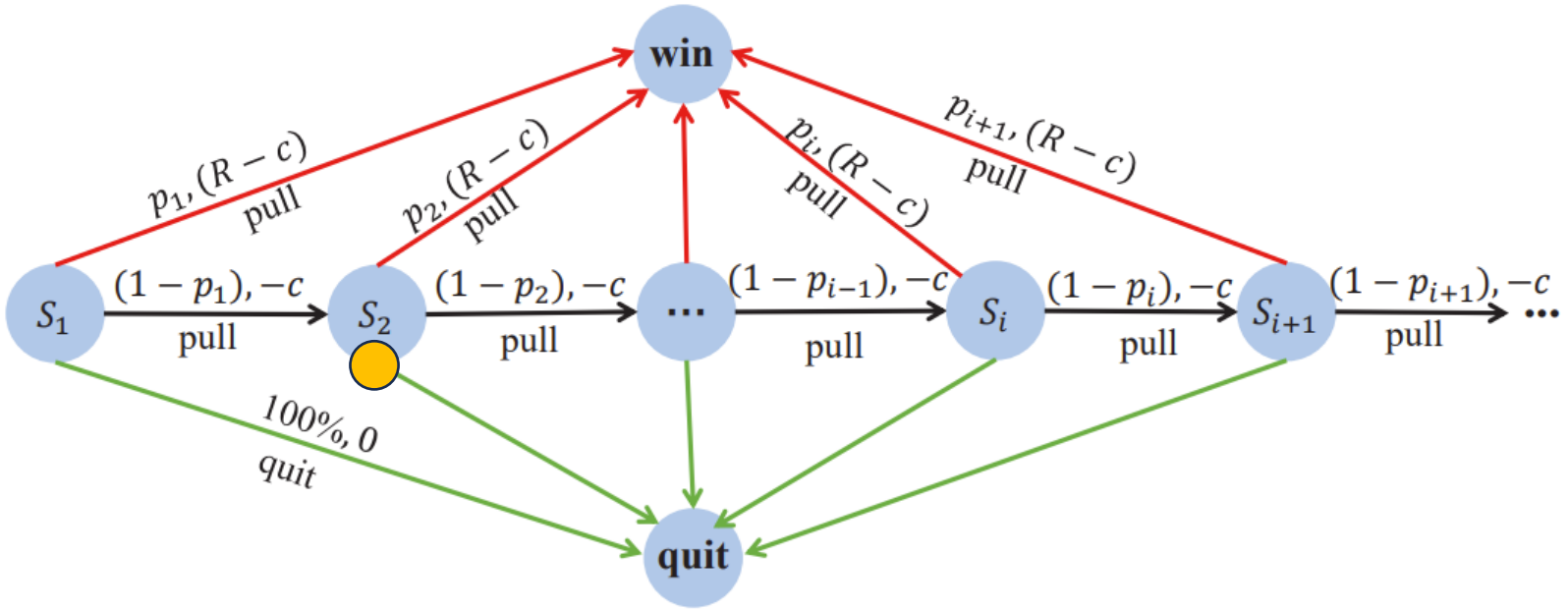
- A seller sells an item to a buyer using gacha game
- The seller will first set up the game configuration:
  - The winning probability of the i-th gacha pull is p_i
  - The price of each gacha pull is c
- The buyer's valuation is R, which is drawn from a distribution F
- The buyer knows p_i and c , and wants to maximize its utility
- The seller knows F and wants to maximize its revenue

Markov Decision Process (Buyer's Perspective)

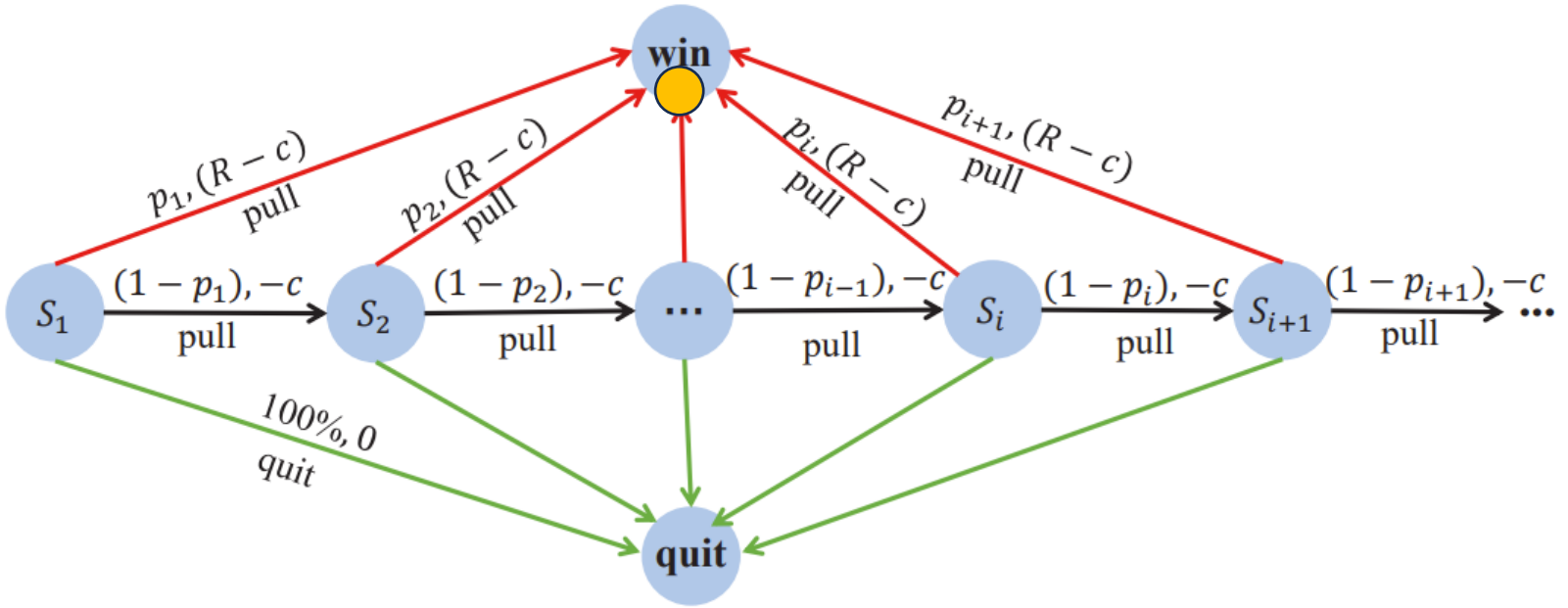


Binary action: Pull or Quit

Markov Decision Process (Buyer's Perspective)



Markov Decision Process (Buyer's Perspective)



π_k denotes the policy that the buyer will pull at most k gacha pulls.

Seller’s Revenue

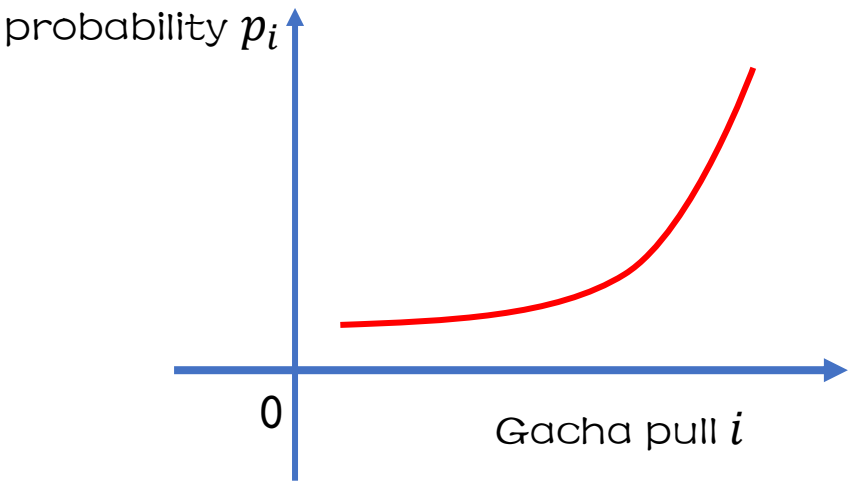
- The seller’s revenue comes from selling the gacha pull.
- Suppose that the buyer adopts the policy π_k
- Then the seller’s revenue is $c \cdot E(\pi_k)$
- $E(\pi_k)$ is the expected number of gacha pulls the buyer will buy.
- c denotes the price of each gacha pull

Whale Property Gacha Game

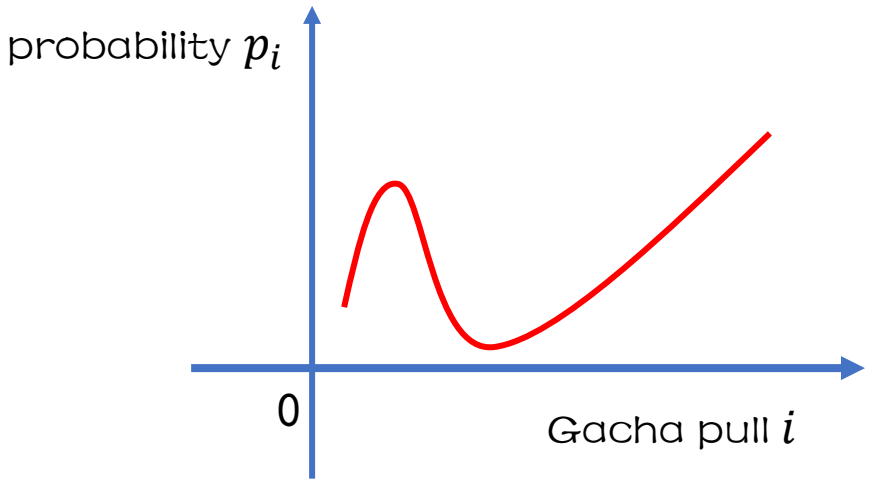
- For a Gacha Game with whale property:
  - The buyer will continue pulling until wins
  - Or the buyer will never pull the gacha game.

- **Whale property ⇔ Take it or Leave it Strategy**

Whale Property Gacha Game



(1) whale property



(2) non−whale property

Gacha Game and Auction

- Gacha game ⇔ single-item single-buyer probabilistic selling
- Whale property ⇔ take-it-or-leave-it strategy
- Gacha game ⇔ single-item single-bidder auction ?

Gacha Game and Auction

- Single-item single-bidder auction:
- Allocation rule: $x(b)$
- Payment rule: $y(b)$
- When a bidder proposes a bid b, he needs to pay $y(b)$ and can get the item with probability $x(b)$
- The utility of the bidder with personal valuation R is

$$u(b) = x(b) \cdot R - y(b)$$

- Using Myerson's Lemma, we can design a DSIC auction such that the bidder will honestly bid. $b \equiv R$

Gacha Game and Auction

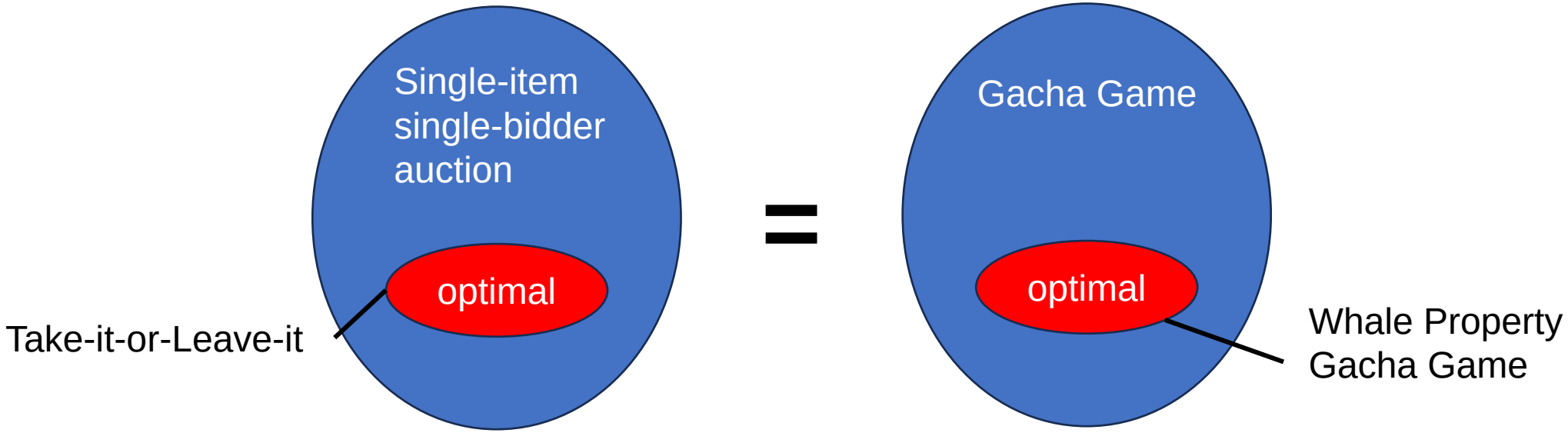
- Gacha Game:
- Buyer with valuation R ⇔ Bidder
- The optimal gacha pulling policy is denoted as $\pi_{opt(R)}$ ⇔ Bidding
- The buyer needs to pay $E(\pi_{opt(R)}) \cdot c$ ⇔ Payment Rule
- The probability of winning the gacha game ⇔ Allocation Rule

$$1 - \prod_{j=1}^{opt(R)} (1 - p_j)$$

- With Myerson's Lemma, Gacha Game is also DSIC

Gacha Game and Auction

- Gacha Game ⇔ Single-item Single-bidder Auction
- Optimal Gacha Game ⇔ Optimal Single-item Single-bidder Auction
- Take-it-or-Leave-it is optimal in auction ⇔ Whale Property is revenue optimal in gacha game



Necessary Condition for Revenue Optimality in Gacha Game

Maximum revenue of **whale property gacha game** > Maximum revenue of **non-whale property gacha game**

Revenue Optimality of whale property gacha game

Advantage of Gacha Selling

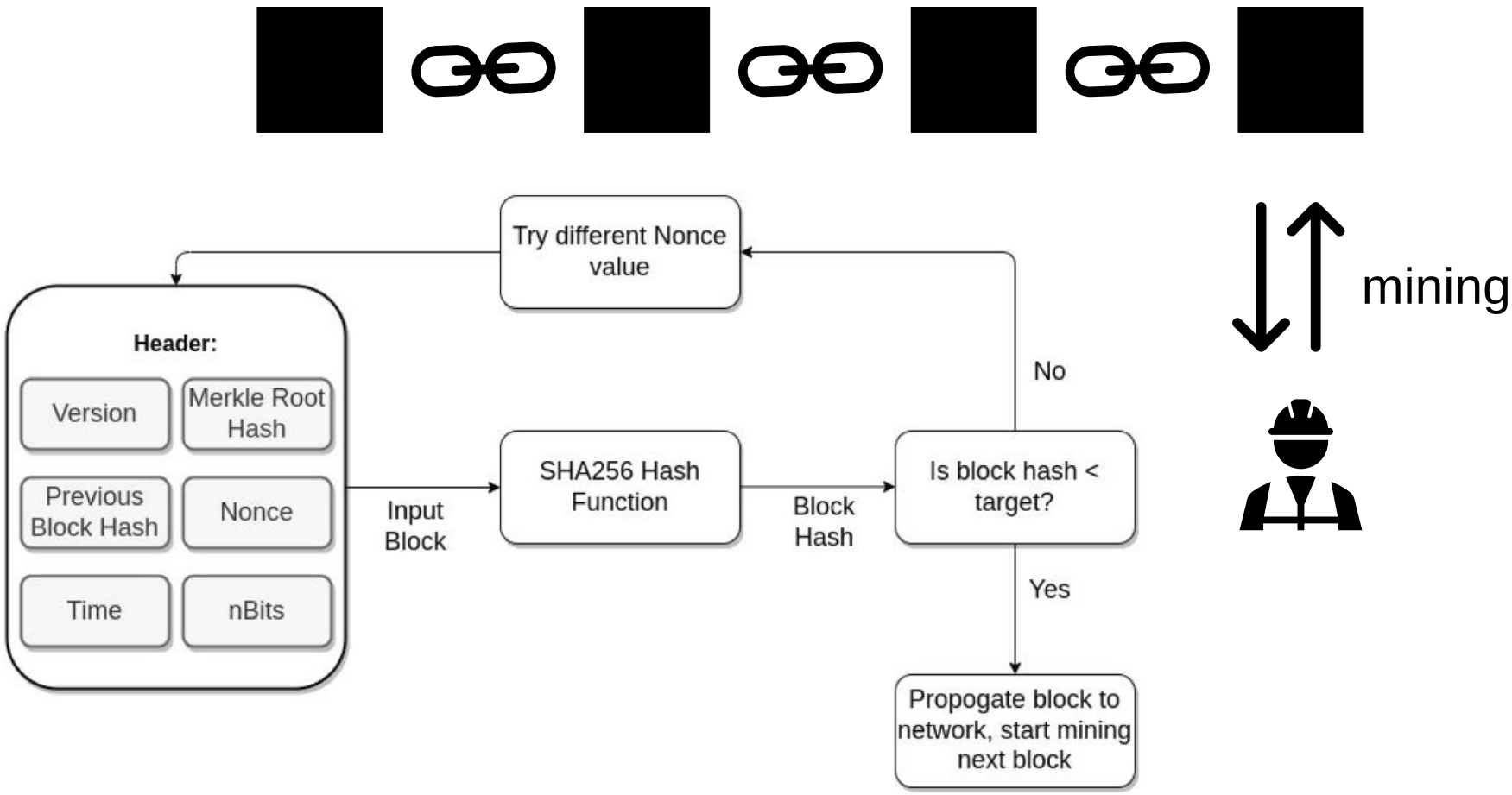
- *With budget constraints, the gacha game can achieve a higher revenue than the take-it-or-leave-it strategy in auction*

- Example:
- Valuation of a buyer $R = 100$
- Budget B follows the distribution $P(B = 50) = 0.5, P(B = 100) = 0.5$
- Gacha game: $p_i = 0.01, c = 1$
- Maximum revenue in auction: 50
- Revenue in Gacha game: 51.448

Multi-item Gacha Game

- Multi-item gacha game includes **multiple phases**
- The buyer can play the gacha game in each phase. Once he wins, he can obtain the reward of the gacha game in that phase.
- We consider two types of the multi-item gacha games:
  - **Sequential gacha game**: will end a phase and enter the next phase only when the buyer wins the gacha game once.
  - **Banner-based gacha game**: allows buyer's opt-out, and will end a phase and enter the next phase when the buyer wins the gacha game once or chooses to opt-out.

Blockchain as a Gacha Game

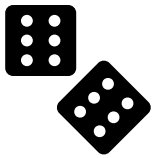


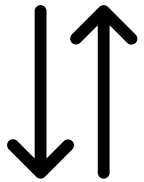
Blockchain as a Gacha Game

System designer
⇔Seller in Gacha

Winning probability
of each gacha pull⇔

Generating block
with a certain probability





Invest computing power
⇔ Price of gacha
pull in Gacha

Block reward
⇔Gacha reward

Miner in blockchain ⇔ Buyer in Gacha

Blockchain as a Gacha Game

Gacha Game	Blockchain	
	PoW Blockchain	PoS Blockchain
buyer	miner	validator
seller	system designer	
gacha reward	block reward	
gacha pull	hash operation with nonce	hash operation with time
winning probability	probability that hash value hits the target	
price of each gacha pull	computing cost for hash operation	staking cost at time
seller's revenue	system's security guarantee	
	invested computing power	invested coins
optimal configuration	mining difficulty adjustment	
gacha game type	fixed-probability gacha game	gacha games in Table 2

Table 1. Blockchain as a gacha game

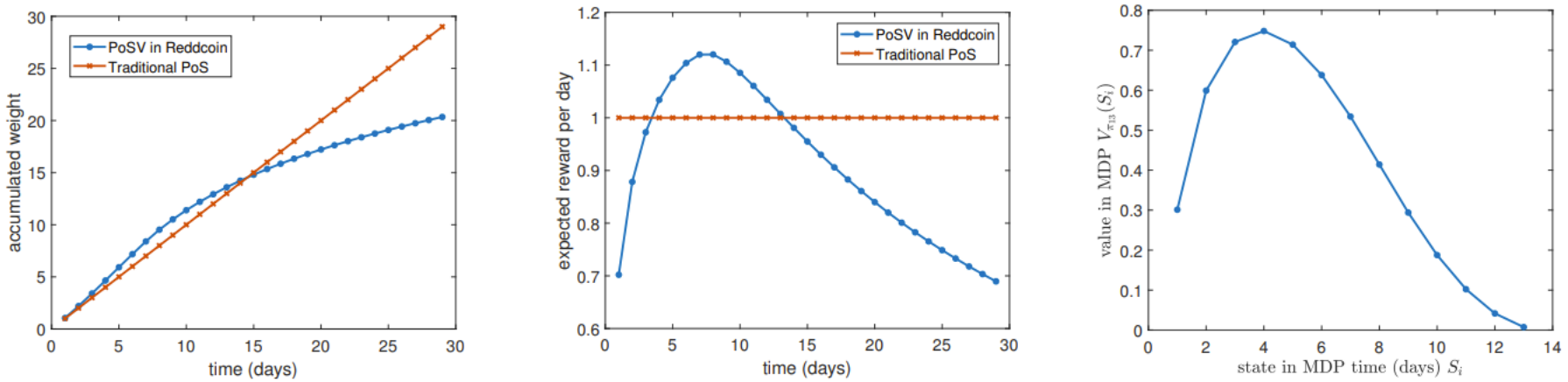
Proof-of-Stake Blockchain

Gacha Game	coins as stake	coin age as stake	
		linear coin age	Reddcoin (PoSV)
gacha game type	fixed-probability gacha game	gacha game with increasing probability	non-whale property gacha game
whale property	✓	✓	✗
reset-after-winning mechanism in sequential gacha game	✗	coin age resets to 0 when the validator finds a new block	
succeed-after-opt-out mechanism in banner-based gacha game	✗	coin age does not reset to 0 when others find a new block	

Table 2. PoS blockchain as a gacha game

Case Study: Reddcoin

$$\text{weight} = \begin{cases} -0.00408163 * \text{time}^3 + 0.05714286 * \text{time}^2 + \text{time}, & \text{time} \leq 7\text{days}, \\ 8.4 * \log(\text{time}) - 7.94564525, & \text{otherwise}. \end{cases}$$



(a) Accumulated weight of the coin age in Reddcoin and traditional PoS

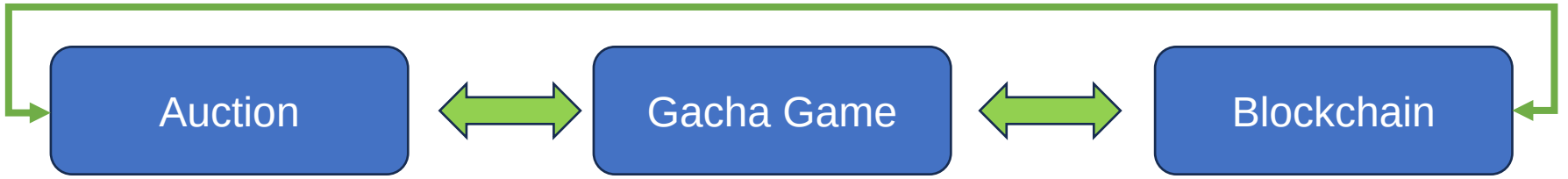
(b) Expected reward per day in Reddcoin and traditional PoS

(c) Value $V_{\pi_{13}}(S_i)$ in MDP of the gacha game model of Reddcoin

Fig. 3. Gacha game model of Reddcoin

Conclusion

- The gacha game is equivalent to the single-item single-bidder auction.
- The whale property gacha game is revenue optimal.
- Blockchain system can be modeled as a gacha game.



- Limitations: One-buyer scenario
- Future work: Multi-buyer Gacha game ⇔ Multi-bidder auction

Thanks~

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